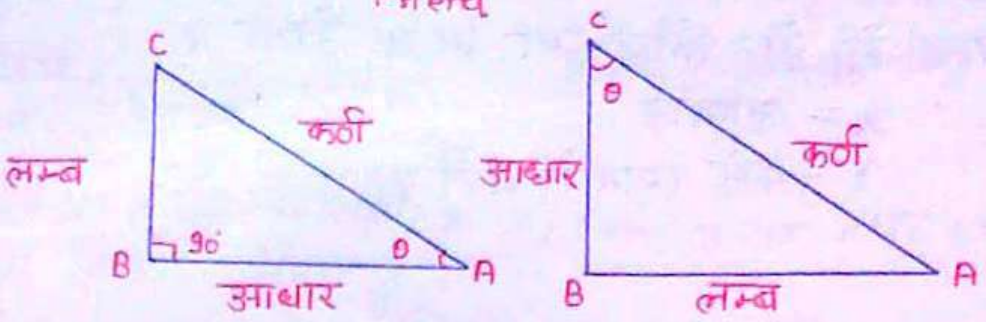
Trignometry :-

त्रिकोणमितीय अनुपात

Friend

bro { Sinθ, Cosθ } bro { tanθ, cotθ } bro { Secθ, CosecΘ }

Friend



पाइथागोरस प्रमेय से —

$(\text{लम्ब})^2 + (\text{आधार})^2 = (\text{कर्ण})^2$

L = लम्ब = Perpendicular = P

A = आधार = Base = B

K = कर्ण = Hypotenuse = H

sinθ →	L	A	L	← tanθ
cosecθ →	K	K	A	← cotθ

$\sin\theta = \frac{\text{लम्ब}}{\text{कर्ण}} \quad \frac{L}{K} < 1$

$\cos\theta = \frac{\text{आधार}}{\text{कर्ण}} \quad \frac{A}{K} < 1$

$\tan\theta = \frac{\text{लम्ब}}{\text{आधार}} \quad \frac{L}{A} \rightarrow$ 1 से कम या 1 या एक से अधिक

$\cot\theta = \frac{\text{आधार}}{\text{लम्ब}} \rightarrow$ 1 से कम या 1 या 1 से अधिक

$\sec\theta = \frac{\text{कर्ण}}{\text{आधार}} > 1$

$\text{Cosec}\theta = \frac{\text{कर्ण}}{\text{लम्ब}} > 1$

Ankur yadav

VISIT…

यदि $\sin\theta = \frac{x}{y}$ तो निम्न में से कौन सा सत्य होगा ?

(a) $x < y$ [✓] ∵ x = लम्ब
(b) $x > y$ y = कर्ण (सबसे बड़ी भुजा)
(c) $x \geq y$
(d) $x \leq y$
(e) $x = y$

यदि $\cos\theta = \frac{x}{y}$ तो निम्न में से कौन सा सत्य होगा ?

(a) $x < y$ [✓] ∵ x = आधार
(b) ~~⟹~~ $x > y$ y = कर्ण (सबसे बड़ी भुजा)
(c) $x \geq y$
(d) $x \leq y$
(e) $x = y$

यदि $\sec\theta = \frac{x}{y}$ तो निम्न में से कौन सा सत्य होगा ?

(a) $x < y$
(b) ~~⟹~~ $x > y$ [✓] ∵ x = कर्ण (सबसे बड़ी भुजा)
(c) $x \geq y$ y = आधार
(d) $x \leq y$
(e) $x = y$

यदि $\text{cosec}\theta = \frac{x}{y}$ तो निम्न में से कौन सा सत्य होगा ?

(a) $x < y$
(b) $x > y$ [✓] ∵ x = कर्ण (सबसे बड़ी भुजा)
(c) $x \geq y$ y = लम्ब
(d) $x \leq y$
(e) $x = y$

समकोण △ की भुजाएँ →

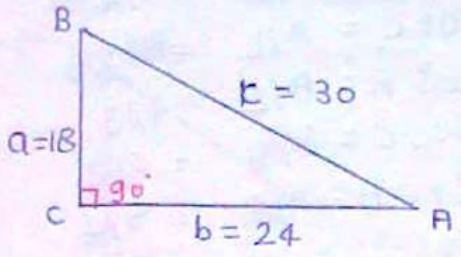
V imp

3, 4, 5
5, 12, 13
6, 8, 10
7, 24, 25
8, 15, 17
9, 40, 41
20, 21, 29

Ankur yadav

Q→ यदि △ ABC में भुजा $a = 18$, $b = 24$, $C = 30$ हो तो $\sin C = ?$

(a) 0 (b) 1 [✓] (c) $1/2$ (d) $\sqrt{3}/2$

S- 

$6 \times (3, 4, 5)$ — एक समकोण $\quad 18, 24, 30$

$\sin C = \sin 90°$

$= 1$

Q→ समकोण △ की भुजाएँ $(x-2)$ cm, x cm तथा $(x+2)$ cm हो तो x का मान कितना होगा ?

(a) 4 (b) 6 (c) 8 [✓] (d) 10

S- लम्ब $= x-2$

आधार $= x$

कर्ण $= x+2$

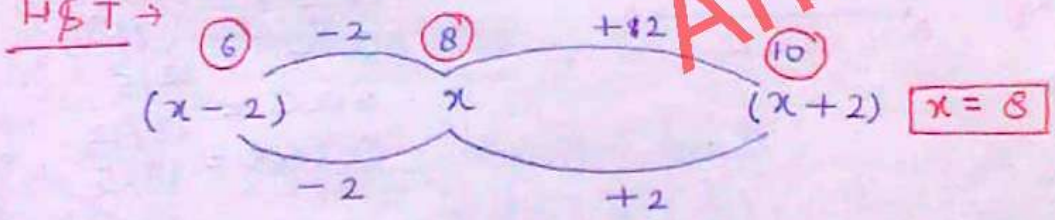
$L^2 + A^2 = K^2$

$(x-2)^2 + (x)^2 = (x+2)^2$

$(x)^2 = (x+2)^2 - (x-2)^2$

$x^2 = 4 \cdot x \cdot 2$

$x = 8$ cm

HBT → 

Q→ किसी समकोण △ की भुजाएँ समान्तर श्रेणी में हैं तो भुजाओं में अनुपात होगा ?

(a) 1 : 2 : 3 (b) 2 : 3 : 4 (c) 3 : 4 : 5 [✓] (d) 4 : 5 : 6

S→ माना समकोण △ की भुजाएँ $(a-d)$, a, $(a+d)$ हैं जो कि समान्तर श्रेणी में हैं

लम्ब $= a-d$

आधार $= a$

कर्ण $= a+d$

(लम्ब)^2 + (आधार)^2 = (कर्ण)^2

$(a-d)^2 + a^2 = (a+d)^2$

$a^2 = (a+d)^2 - (a-d)^2$

$a^2 = 4 \cdot a \cdot d$

$a = 4d$

भुजाओं में अनुपात

$(a-d) : a : (a+d)$

$(4d-d) : 4d : (4d+d)$

$3d : 4d : 5d$

$3 : 4 : 5$

Ankur yadav

Q→ यदि △ ABC में भुजा $a = 18$, $b = 24$, $C = 30$ हो तो $\sin C = ?$

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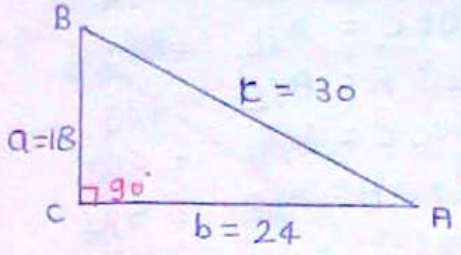
S- 

$6 \times (3, 4, 5)$ — एक समकोण $\quad 18, 24, 30$

$\sin C = \sin 90°$

$= 1$

Q→ समकोण △ की भुजाएँ $(x-2)$ cm, x cm तथा $(x+2)$ cm हो तो x का मान कितना होगा ?

(a) 4 (b) 6 (c) 8 [✓] (d) 10

S- लम्ब $= x-2$

आधार $= x$

कर्ण $= x+2$

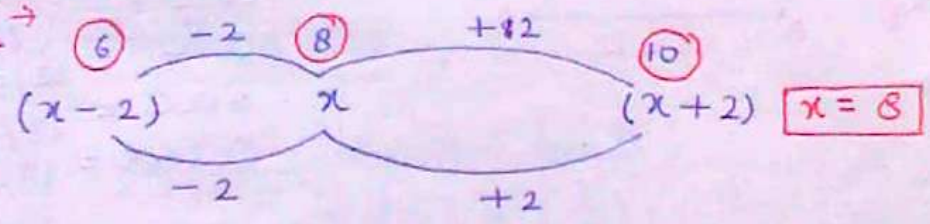
$L^2 + A^2 = K^2$

$(x-2)^2 + (x)^2 = (x+2)^2$

$(x)^2 = (x+2)^2 - (x-2)^2$

$x^2 = 4 . x . 2$

$x = 8$ cm

HBT → 

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(a) 1 : 2 : 3 (b) 2 : 3 : 4 (c) 3 : 4 : 5 [✓] (d) 4 : 5 : 6

S→ माना समकोण △ की भुजाएँ $(a-d)$, a, $(a+d)$ हैं जो कि समान्तर श्रेणी में हैं

लम्ब $= a-d$

आधार $= a$

कर्ण $= a+d$

(लम्ब)^2 + (आधार)^2 = (कर्ण)^2

$(a-d)^2 + a^2 = (a+d)^2$

$a^2 = (a+d)^2 - (a-d)^2$

$a^2 = 4 . a . d$

$a = 4d$

भुजाओं में अनुपात

$(a-d) : a : (a+d)$

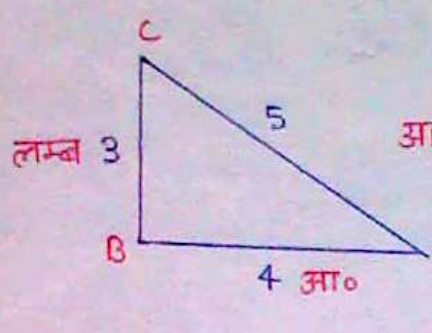
$(4d-d) : 4d : (4d+d)$

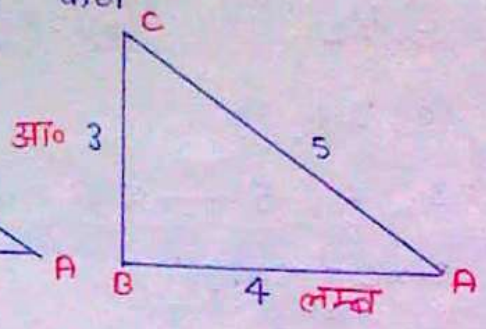
$3d : 4d : 5d$

$3 : 4 : 5$

Q→ यदि $\triangle ABC$ में $\angle B = 90$ तथा $\sin A = \frac{3}{5}$ तो

S→ $\sin A = \frac{3}{5} = \frac{\text{लम्ब}}{\text{कर्ण}}$

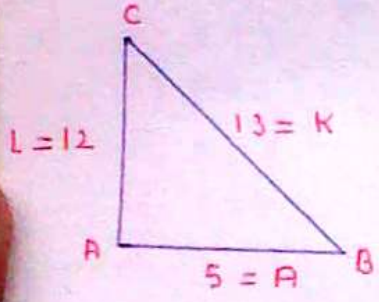


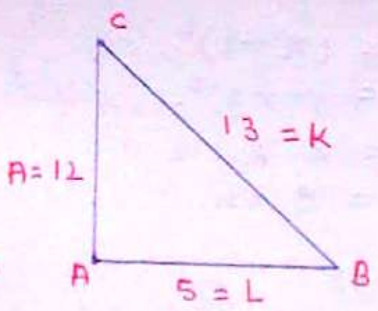


$\cos C = A/K = 3/5$
$\cos A = A/K = 4/5$
$\tan C = L/A = 4/3$
$\tan A = L/A = 3/4$
$\cot C = A/L = 3/4$
$\cot A = A/L = 4/3$
$\sec C = K/A = 5/3$
$\sec A = K/A = 5/4$
$\text{Cosec } C = K/L = 5/4$
$\text{Cosec } A = K/L = 5/3$

Q→ यदि $\triangle ABC$ में $\angle A = 90$ तथा $\cos B = \frac{5}{13}$ तो

S→ $\cos B = \frac{5}{13} = \frac{\text{आ०}}{\text{कर्ण}}$

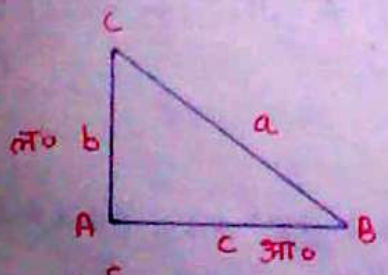


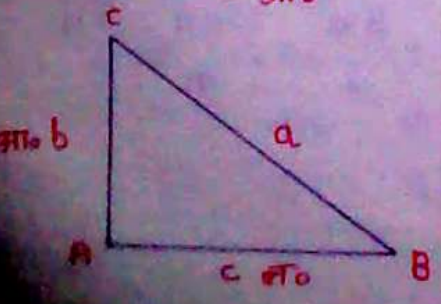


$\sin B = 12/13$
$\tan B = 12/5$
$\cot B = 5/12$
$\sec B = 13/5$
$\text{Cosec } B = 13/12$
$\sin C = 5/13$
$\tan C = 5/12$
$\cos C = 12/13$
$\cot C = 12/5$
$\sec C = 13/12$
$\text{Cosec } C = 13/5$

Q→ $\triangle ABC$ में $\angle A = 90$ $AB = c$, $BC = a$, $CA = b$
तो $\tan B + \tan C = ?$

(a) $\frac{a^2}{bc}$ [✓] (b) $\frac{b^2}{ca}$ (c) $\frac{c^2}{ab}$ (d) $\frac{a^2 - b^2}{a^2 + b^2}$

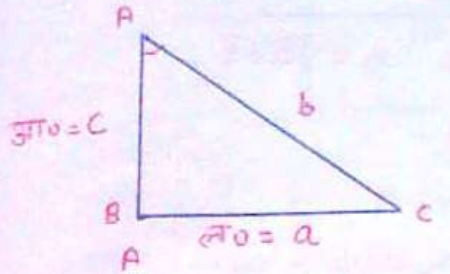


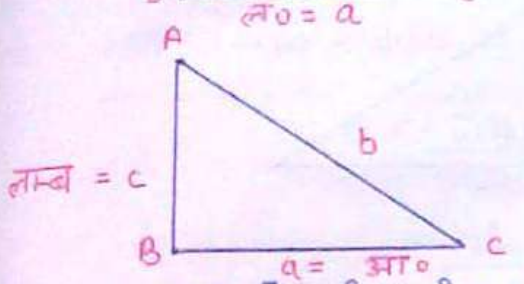


$$\tan B + \tan C$$
$$= \left(\frac{L}{A}\right) + \left(\frac{L}{A}\right)$$
$$= \left(\frac{b}{c}\right) + \left(\frac{c}{b}\right)$$
$$= \frac{b^2 + c^2}{bc}$$
$$= \frac{a^2}{bc}$$

Q→ $\triangle ABC$ में $\angle B = 90^\circ$ तथा $AB = c$, $BC = a$, $CA = b$
तो $\cot A + \cot C = ?$

ⓐ $\frac{a^2}{bc}$ ⓑ $\frac{b^2}{ca}$ ⓒ $\frac{c^2}{ab}$ ⓓ $\frac{b}{ca}$





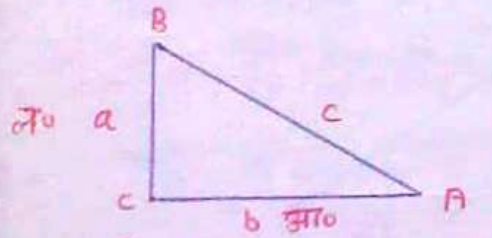
$$\cot A + \cot C$$
$$\left(\frac{A}{L}\right) + \left(\frac{A}{L}\right)$$
$$= \left(\frac{c}{a}\right) + \left(\frac{a}{c}\right)$$
$$= \frac{c^2 + a^2}{ac}$$
$$= \frac{b^2}{ca}$$

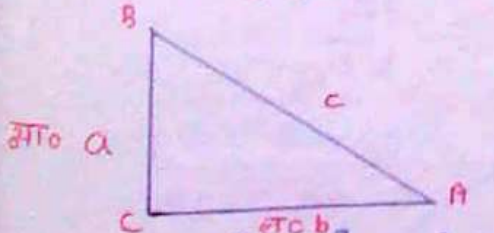
Q→ $\triangle ABC$ में $c^2 = a^2 + b^2$
तो $\tan A + \tan B = ?$

ⓐ $\frac{a^2}{bc}$ ⓑ $\frac{b^2}{ca}$ ⓒ $\frac{c^2}{ab}$ ⓓ $\frac{a+b}{ab}$

$\because c^2 = a^2 + b^2$
$\therefore \triangle ABC$ समकोण $\triangle$ होगा

Ankur yadav



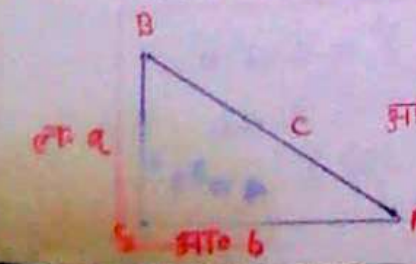


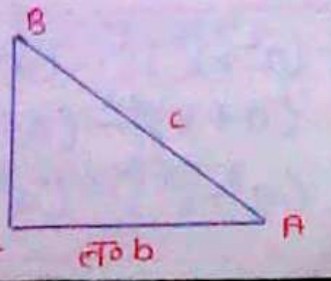
$$\tan A + \tan B$$
$$= \left(\frac{L}{A}\right) + \left(\frac{L}{A}\right)$$
$$= \left(\frac{a}{b}\right) + \left(\frac{b}{a}\right)$$
$$= \frac{a^2 + b^2}{ba}$$
$$= \frac{c^2}{ab}$$

Q→ $\triangle ABC$ में, $C^2 = a^2 + b^2$ तो $\cot A + \cot B = ?$

ⓐ $\frac{a^2}{bc}$ ⓑ $\frac{b^2}{ca}$ ⓒ $\frac{c^2}{ab}$ ⓓ $\frac{a+b}{ab}$

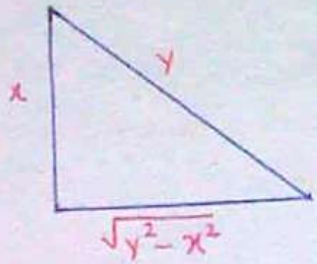
$\because c^2 = a^2 + b^2$
$\therefore \triangle ABC$ समकोण $\triangle$ होगा

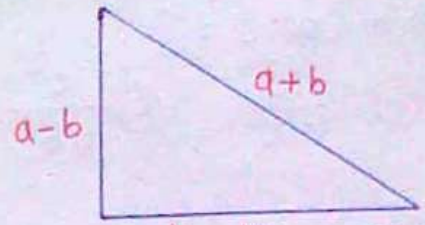


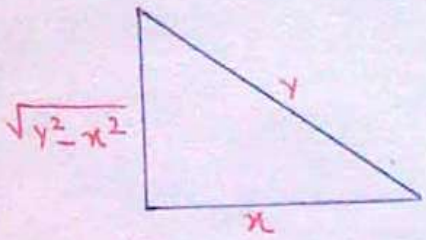


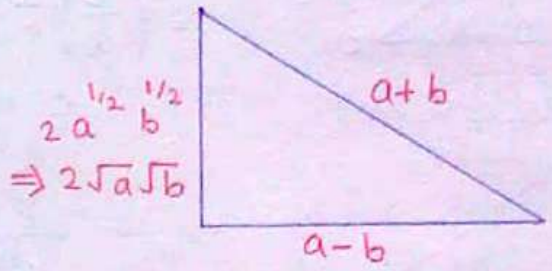
$$\cot A + \cot B$$
$$\left(\frac{A}{L}\right) + \left(\frac{A}{L}\right)$$
$$= \left(\frac{b}{a}\right) + \left(\frac{a}{b}\right)$$
$$= \frac{b^2 + a^2}{ab}$$
$$= \frac{c^2}{ab}$$

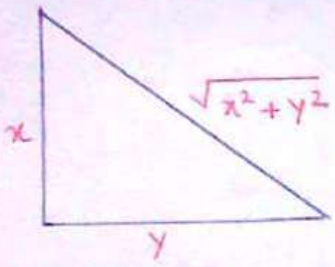
NOTE →

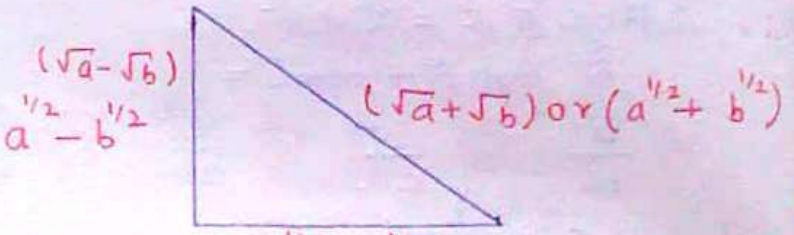


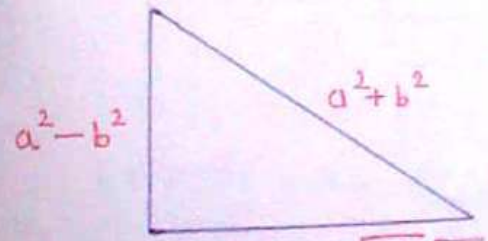


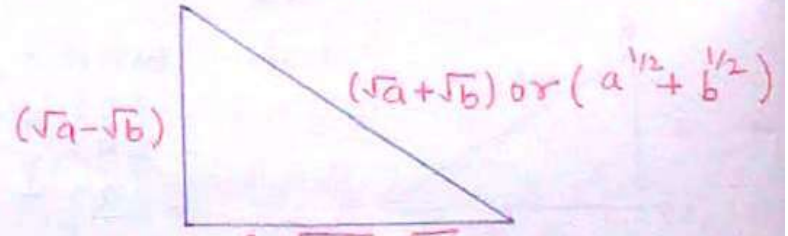


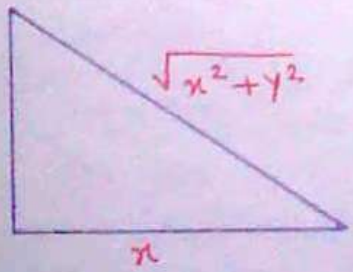


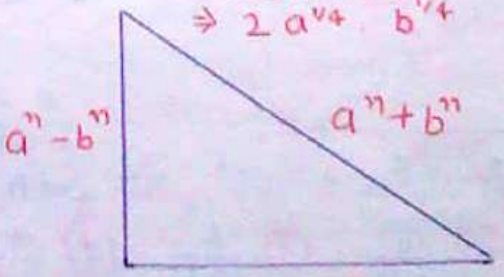


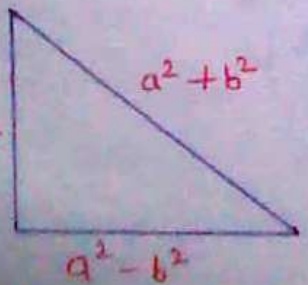












$(a^2+b^2)^2-(a^2-b^2)^2 = 4a^2b^2$

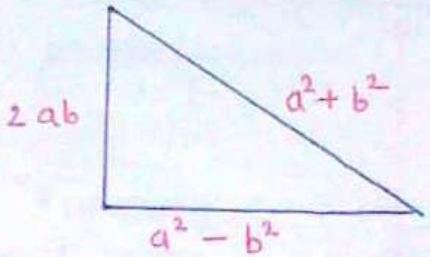
$(a+b)^2-(a-b)^2 = 4ab$

$(a^3+b^3)^2-(a^3-b^3)^2 = 4a^3b^3$

Ankur yadav

Q→ यदि $(a^2-b^2)\cos\theta + 2ab\sin\theta = a^2+b^2$ तो $\tan\theta = ?$

आ० आ० लं० लं० कर्ण



$$\tan\theta = \frac{2ab}{a^2-b^2}$$

Q→ यदि $(a^2-b^2)\sin\theta + 2ab\cos\theta = a^2+b^2$
तो $\tan\theta = ?$

$$\tan\theta = \frac{\sin\theta \text{ का गुणांक}}{\cos\theta \text{ का गुणांक}}$$

$$= \frac{a^2-b^2}{2ab}$$

Trick→ यदि $p\sin\theta + q\cos\theta = \sqrt{p^2+q^2}$
तो $\tan\theta = \frac{\sin\theta \text{ का गुणांक}}{\cos\theta \text{ का गुणांक}} = \frac{p}{q}$

Q- यदि $5\sin\theta + 12\cos\theta = 13$
तो $\tan\theta = \frac{5}{12}$

Q→ यदि $9\cos\theta + 40\sin\theta = 41$
तो $\tan\theta = \frac{40}{9}$

$\sin^2\theta + \cos^2\theta = 1$

NOTE:→

$\sin^2(2\theta) + \cos^2(2\theta) = 1$
$\sin^2(3\theta) + \cos^2(3\theta) = 1$
$\sin^2(ax+b) + \cos^2(ax+b) = 1$

समान कोण

$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

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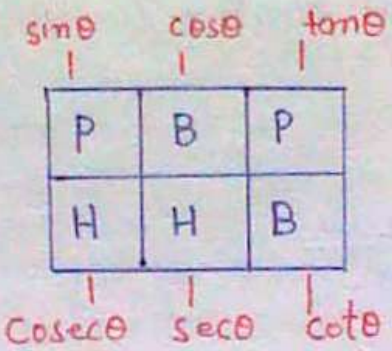

$\sin\theta = \frac{1}{\text{cosec}\theta}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

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$\sin\theta \longrightarrow \tan\theta \longrightarrow \sec\theta$

$\cos\theta \longrightarrow \cot\theta \longrightarrow \text{cosec}\theta$

$\sin\theta = \frac{\tan\theta}{\sec\theta} \rightarrow \frac{1}{\sin\theta} = \text{cosec}\theta = \frac{\sec\theta}{\tan\theta}$

$\cos\theta = \frac{\cot\theta}{\text{cosec}\theta} \rightarrow \frac{1}{\cos\theta} = \sec\theta = \frac{\text{cosec}\theta}{\cot\theta}$

$\tan\theta = \frac{\sec\theta}{\text{cosec}\theta}$

$\cot\theta = \frac{\text{cosec}\theta}{\sec\theta}$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\sin\theta = \frac{\tan\theta}{\sec\theta}$$

$$\sin\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}}$$

$$\sin\theta = \frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{1}{\sqrt{\sec^2\theta-1}}$$

$$\cot\theta = \sqrt{\operatorname{cosec}^2\theta-1}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$\cos\theta = \frac{\cot\theta}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\cos\theta = \frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec\theta = \frac{1}{\sqrt{1-\sin^2\theta}}$$

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$\sec\theta = \frac{\operatorname{cosec}\theta}{\cot\theta}$$

$$\sec\theta = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$$

$$\sec\theta = \frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$\tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\tan\theta = \sqrt{\sec^2\theta-1}$$

$$\tan\theta = \frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sqrt{1-\cos^2\theta}}$$

$$\operatorname{cosec}\theta = \frac{\sec\theta}{\tan\theta}$$

$$\operatorname{cosec}\theta = \frac{\sqrt{1+\tan\theta}}{\tan\theta}$$

$$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$$

$$\operatorname{cosec}\theta = \frac{\sec\theta}{\sqrt{\sin^2\theta-1}}$$

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$\sin^2\theta + \cos^2\theta = 1$

(1) $\sin^2\theta = 1 - \cos^2\theta$

(2) $\sin\theta = \sqrt{1-\cos^2\theta}$

(3) $\sin\theta = \sqrt{1+\cos\theta\,(1-\cos\theta)}$

(4) $\sin^2\theta = (1+\cos\theta)(1-\cos\theta)$

(5) $\dfrac{\sin^2\theta}{1+\cos\theta} = 1-\cos\theta$

(6) $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

(7) $\dfrac{1}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin^2\theta} = \operatorname{cosec}^2\theta + \operatorname{cosec}\theta\cdot\cot\theta$

(8) $\dfrac{1}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin^2\theta} = \operatorname{cosec}^2\theta - \operatorname{cosec}\theta\cdot\cot\theta$

(9) $\sin\theta\cdot\sin\theta = (1+\cos\theta)(1-\cos\theta)$

(10) $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta} = \operatorname{cosec}\theta - \cot\theta$

(11) $\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin\theta} = \operatorname{cosec}\theta + \cot\theta$

Q → $1 - \dfrac{\sin^2\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{1-\cos\theta} = ?$

(i) 0 (ii) 1 (iii) $\sin\theta$ (iv) $\cos\theta$

S- Ist

$$\left(1 - \frac{\sin^2\theta}{1+\cos\theta}\right) + \left(\frac{1+\cos\theta}{\sin\theta} - \frac{\sin\theta}{1-\cos\theta}\right) = ?$$

$$= \frac{1+\cos\theta-\sin^2\theta}{1+\cos\theta} + \left(\frac{(1+\cos\theta)(1-\cos\theta)-\sin^2\theta}{\sin\theta\,(1-\cos\theta)}\right)$$

$$= \frac{1+\cos\theta-\sin^2\theta}{(1+\cos\theta)} + \left(\frac{1-\cos^2\theta-\sin^2\theta}{\sin\theta\,(1-\cos\theta)}\right)$$

$$= \frac{\cos\theta+\cos^2\theta}{1+\cos\theta} + \left(\frac{1-(\cos^2\theta+\sin^2\theta}{\sin\theta\,(1-\cos\theta)}\right)$$

$$= \frac{\cos\theta(1+\cos\theta)}{1+\cos\theta} + \left(\frac{1-1}{\sin\theta\,(1-\cos\theta)}\right)$$

$$= \cos\theta + \frac{1-1}{\sin\theta\,(1-\cos\theta)} \quad \frac{0}{\sin\theta\,(1-\cos\theta)}$$

$$= \cos\theta + 0$$

$\cos\theta$ ANS-

IInd $\left(1-\frac{\sin^2\theta}{1+\cos\theta}\right)+\left(\frac{1+\cos\theta}{\sin\theta}-\frac{\sin\theta}{1-\cos\theta}\right)=?$

$=\left[1-(1-\cos\theta)\right]+\left[\frac{1+\cos\theta}{\sin\theta}-\frac{(1-\cos\theta)}{\sin\theta}\right]$

$=[1-1+\cos\theta]+[0]$

$=\cos\theta$ ANS.

(12) $\cos^2\theta = 1-\sin^2\theta$

(13) $\cos\theta = \sqrt{1-\sin^2\theta}$

(14) $\cos\theta = \sqrt{(1+\sin\theta)(1-\sin\theta)}$

(15) $\cos^2\theta = (1+\sin\theta)(1-\sin\theta)$

(16) $\frac{\cos^2\theta}{1+\sin\theta} = 1-\sin\theta$

(17) $\frac{\cos^2\theta}{1-\sin\theta} = 1+\sin\theta$

(18) $\frac{1}{1-\sin\theta} = \frac{1+\sin\theta}{\cos^2\theta} = \sec^2\theta+\sec\theta\cdot\tan\theta$

(19) $\frac{1}{1+\sin\theta} = \frac{1-\sin\theta}{\cos^2\theta} = \sec^2\theta-\sec\theta\cdot\tan\theta$

(20) $\cos\theta\cdot\cos\theta = (1+\sin\theta)(1-\sin\theta)$

(21) $\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta} = \sec\theta-\tan\theta$

(22) $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta} = \sec\theta+\tan\theta$

(23) $\sin^2\theta-1 = -\cos^2\theta$

(24) $(\sin\theta+1)(\sin\theta-1) = -\cos^2\theta$

(25) $\cos^2\theta-1 = -\sin^2\theta$

(26) $(\cos\theta+1)(\cos\theta-1) = -\sin^2\theta$

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Q. Ist $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = ?$

$= \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A}$

$= \frac{\sin^2 A + (1^2 + \cos^2\theta + 2\cdot 1\cdot\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta)\cdot\sin\theta}$

$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{(1+\cos\theta)\cdot\sin\theta}$

$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\cdot\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\,\text{Cosec}\,\theta$

IInd $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$

$= \frac{1-\cos A}{\sin A} + \frac{1+\cos A}{\sin A}$

$= \frac{1-\cos A + 1 + \cos A}{\sin A}$

$= \frac{2}{\sin A}$

$= 2\,\text{Cosec}\,A$

IIIrd $\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$

$= \frac{1-\cos A}{\sin\theta} + \frac{(1+\cos A)}{\sin A}$

$= (\text{Cosec}A - \cot A) + (\text{Cosec}A + \cot A)$

$= 2\,\text{Cosec}A$

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$$\frac{1+\sin A}{\cos A} = \frac{\cos A}{1-\sin A}$$

$$\frac{1-\sin A}{\cos A} = \frac{\cos A}{1+\sin A}$$

$$\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

$$\frac{\sin A}{1-\cos A} = \frac{1+\cos A}{\sin A}$$

$$\frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$$

$$\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$$\frac{\cos\theta}{1+\sin\theta} = \frac{1-\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$$

Questions →

1- $\frac{\sin A}{1+\cos A} - \frac{1+\cos A}{\sin A} =$

2- $\frac{1+\cos A}{\sin A} - \frac{\sin A}{1+\cos\theta} =$

3- $\frac{\sin A}{1-\cos A} + \frac{(1-\cos A)}{\sin A} =$

4- $\frac{\sin A}{1-\cos A} - \frac{(1-\cos A)}{\sin A} =$

5- $\frac{1-\cos A}{\sin A} - \frac{\sin A}{1-\cos A} =$

6- $1 - \frac{\sin^2 A}{1+\cos A} + \frac{1+\cos A}{\sin A} - \frac{\sin A}{1-\cos A} =$

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Q 1→ $\sqrt{\frac{1+\sin A}{1-\sin A}} = ?$

$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$

$= \sqrt{\frac{(1+\sin A)^2}{1^2-\sin^2 A}}$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A}$

$\frac{\cos A}{\sin A}$

Q 2→ $\sqrt{\frac{1-\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}}$

$= \sqrt{\frac{(1-\sin A)^2}{1^2-\sin^2 A}}$

$= \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$

$= \frac{1-\sin A}{\cos A}$

$= \frac{\cos A}{1+\sin A}$

Q3 → $\sqrt{\frac{1+\cos A}{1-\cos A}}$

$= \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$

$= \sqrt{\frac{(1+\cos A)^2}{1^2-\cos^2 A}}$

$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$

$= \frac{1+\cos A}{\sin A}$

$= \frac{\sin A}{1-\cos A}$

Q4 → $\sqrt{\frac{1-\cos A}{1+\cos A}}$

$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$

$= \sqrt{\frac{(1-\cos A)^2}{1^2-\cos^2 A}}$

$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$

$= \sqrt{\frac{1-\cos A}{\sin A}} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \frac{\sin A}{1+\cos A}$

$= \operatorname{cosec} A \cdot \cot A$

Q.5 → $\sqrt{\frac{\operatorname{cosec} A+1}{\operatorname{cosec} A-1}}$

$= \sqrt{\frac{(\operatorname{cosec} A+1)(\operatorname{cosec} A+1)}{(\operatorname{cosec} A-1)(\operatorname{cosec} A+1)}}$

$= \sqrt{\frac{(\operatorname{cosec} A+1)^2}{\operatorname{cosec}^2 A-1}}$

$= \sqrt{\frac{(\operatorname{cosec} A+1)^2}{\cot^2 A}}$

$= \frac{\operatorname{cosec} A+1}{\cot A} = \frac{\operatorname{cosec} A}{\cot A} + \frac{1}{\cot A}$

$= \sec A + \tan A$

1 → $\sqrt{\frac{1+\sin A}{1-\sin A}} = \frac{1+\sin A}{\cos A} = \frac{\cos A}{1-\sin A} = \sec A + \tan A = \sqrt{\frac{\operatorname{cosec} A+1}{\operatorname{cosec} A-1}} = \frac{\operatorname{cosec} A+1}{\cot A} = \frac{\cot A}{\operatorname{cosec} A-1}$

2 → $\sqrt{\frac{1-\sin A}{1+\sin A}} = \frac{1-\sin A}{\cos A} = \frac{\cos A}{1+\sin A} = \sec A - \tan A = \sqrt{\frac{\operatorname{cosec} A-1}{\operatorname{cosec} A+1}} = \frac{\operatorname{cosec} A-1}{\cot A} = \frac{\cot A}{\operatorname{cosec} A+1}$

3 → $\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A} = \operatorname{cosec} A + \cot A = \sqrt{\frac{\sec A+1}{\sec A-1}} = \frac{\sec A+1}{\sec \tan A} = \frac{\tan A}{\sec A-1}$

4 → $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A = \sqrt{\frac{\sec A-1}{\sec A+1}} = \frac{\sec A-1}{\tan A} = \frac{\tan A}{\sec A+1}$

Q→ यदि $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

तो $\cos\theta - \sin\theta = ?$

(a) $2\sin\theta$ (b) $2\cos\theta$ (c) $\sqrt{2}\sin\theta$ (d) $\sqrt{2}\cos\theta$

S→ **Ist method -**

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\cos^2\theta + \sin^2\theta + 2\cos\theta \cdot \sin\theta = 2\cos^2\theta$$

$$2\cos\theta\sin\theta = 2\cos^2\theta - \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - \sin^2\theta$$

$$= (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$$

$$2\sin\theta\cos\theta = (\sqrt{2}\cos\theta)(\cos\theta - \sin\theta)$$

$$\frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta} = \cos\theta - \sin\theta$$

$$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

IInd Method

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta \quad \text{——} (I)$$

$$\cos\theta - \sin\theta = x \text{ माना} \quad \text{——} (II)$$

$$(I)^2 + (II)^2$$

$$(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = (\sqrt{2}\cos\theta)^2 + x^2$$

$$\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta + \cos^2\theta - \sin^2\theta - 2\cos\theta\sin\theta = 2\cos^2\theta + x^2$$

$$1 + 1 = 2\cos^2\theta + x^2$$

$$2 - 2\cos^2\theta = x^2$$

$$2(1 - \cos^2\theta) = x^2$$

$$2\sin^2\theta = x^2$$

$$\sqrt{2}\sin\theta = x$$

$$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

IIIrd Method

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta \quad \text{——} (I)$$

$$\cos\theta - \sin\theta = x \text{ माना} \quad \text{——} (II)$$

$$(I)^2 + (II)^2$$

$$(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = (\sqrt{2}\cos\theta)^2 + x^2$$

$$2(\cos^2\theta + \sin^2\theta) = 2\cos^2\theta + x^2$$

$$2 + 2\sin^2\theta = 2\cos^2\theta + x^2$$

$$2\sin^2\theta = x^2$$

Mission upsc

$\sqrt{2}\sin\theta = x$

$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$

IVth Method -

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\sin\theta = (\sqrt{2}-1)\cos\theta$

$\sin\theta = \dfrac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}\cos\theta$

$\sin\theta(\sqrt{2}+1) = (\sqrt{2})^2 - 1^2)\cos\theta$

$\sqrt{2}\sin\theta + \sin\theta = (2-1)\cos\theta$

$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$

Vth Method - Trick

यदि $a\cos\theta + b\sin\theta = x$

$b\cos\theta - a\sin\theta = y$

तो $a^2+b^2 = x^2+y^2$

$1\cos\theta + 1\sin\theta = \sqrt{2}\cos\theta \quad a=1$

$1\cos\theta + 1\sin\theta = x$ (माना) $\quad b=1$

तो $1^2+1^2 = (\sqrt{2}\cos\theta)^2 + x^2$

$2 = 2\cos^2\theta + x^2$

$2-2\cos^2\theta = x^2$

$2\sin^2\theta = x^2$

$\sqrt{2}\sin\theta = x$

$\sqrt{2}\sin\theta = \cos\theta - \sin\theta$

VIst Method - HBT

(i) यदि $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$
तो $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

(ii) यदि $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$
तो $\cos\theta + \sin\theta = \sqrt{2}\sin\theta$

(iii) यदि $\cos\theta + \sin\theta = \sqrt{2}\sin\theta$
तो $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$

(iv) यदि $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$
तो $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Trick:→

(i) यदि $ax + by = p$
तथा $bx - ay = q$
तो $(a^2+b^2)(x^2+y^2) = p^2+q^2$

(ii) यदि $ax + by = p$
तथा $ay - bx = q$
तो $(a^2+b^2)(x^2+y^2) = p^2+q^2$

(iii) यदि $ax - by = p$
तथा $bx + ay = q$
तो $(a^2+b^2)(x^2+y^2) = p^2+q^2$

(iv) यदि $ax - by = p$
$ay + bx = q$
तो $(a^2+b^2)(x^2+y)^2 = p^2+q^2$

Q→ यदि $2\cos\theta + 3\sin\theta = 3$
तो $3\cos\theta - 2\sin\theta = ?$

$a^2+b^2 = x^2+y^2$

$3\cos\theta - 2\sin\theta = x$

$2^2 + 3^2 = 3^2 + x^2$

$2^2 = x^2$

$\boxed{x = 2}$

Q→ यदि $a\cos\theta + b\sin\theta = c$
तो $b\cos\theta - a\sin\theta = ?$

Mission upsc

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$

$\cot(90-\theta) = \tan\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90-\theta) = \sec\theta$

	0	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

$\cos 0° = \sin 90° = 1$

$\cos 30° = \sin 60° = \sqrt{3}/2$

$\cos 45 = \sin 45° = 1/\sqrt{2}$

$\cos 60 = \sin 30° = 1/2$

$\cos 90° = \sin 0 = 0$

$\sin 10 = \cos 80°$

$\sin 20 = \cos 70°$

$\sin 30 = \cos 60°$

$\sin 40 = \cos 50$

$\sin 50 = \cos 40°$

$\sin 60 = \cos 30°$

$\sin 70 = \cos 20°$

$\sin 80° = \cos 10°$

$\sin 90° = \cos 0$

$\cos 5° = \sin 85°$

$\cos 15 = \sin 75$

$\cos 25 = \sin 65$

$\cos 35 = \sin 55$

$\cos 45 = \sin 45$

$\cos 55 = \sin 35$

$\cos 65 = \sin 25°$

$\cos 75 = \sin 15°$

$\cos 85 = \sin 5°$

$\cos(5-\theta) = \sin(85+\theta)$

$\cos(20-\theta) = \sin(70+\theta)$

$\cos(55-\theta) = \sin(35+\theta)$

$\sin(60+\theta) = \cos(30-\theta)$

$\sin(55-\theta) = \cos(35+\theta)$

$\sin(50+\theta) = \cos(40-\theta)$

$\cos(35-\theta) = \sin(55+\theta)$

$\cos(45-\theta) = \sin(45+\theta)$

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